FORMULARIO VIBRACIONES

Ecuación diferencial: $\boxed{m_e\ddot{x} + c_e\dot{x} + k_e x = f(t)}$

Adimensional: $\boxed{\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = f(t)/m}$

Respuesta: $\boxed{x(t) = x_h(t) + x_p(t)}$

con : $\boxed{\omega_n = \dfrac{2\pi}{\tau_n}}$ $\boxed{\omega_n = 2\pi f_n}$ $\boxed{\omega_n = \sqrt{\dfrac{k_e}{m_e}}}$ $\boxed{\zeta = \dfrac{c_e}{2\sqrt{m_e k_e}}}$ $\boxed{\omega_a = \omega_n\sqrt{1-\zeta^2}}$

VIBRACIONES LIBRES: $x(t) = x_h(t)$

Sistema no amortiguado: $\zeta = 0$

$$x(t) = A\,\mathbf{sen}(\omega_n t) + B\,\mathbf{cos}(\omega_n t) \qquad A = \frac{\dot{x}(0)}{\omega_n} \qquad B = x(0)$$

$$x(t) = X_o\,\mathbf{cos}(\omega_n t - \phi) \qquad X_o = \sqrt{\left(\frac{\dot{x}(0)}{\omega_n}\right)^2 + \left(x(0)\right)^2} \qquad \phi = \mathbf{tan}^{-1}\frac{\dot{x}(0)}{x(0)\omega_n}$$

Sistema subamortiguado: $0 < \zeta < 1$

$$x(t) = e^{-\zeta\omega_n t}\left(A\,\mathbf{sen}(\omega_a t) + B\,\mathbf{cos}(\omega_a t)\right) \qquad A = \frac{\dot{x}(0) + \zeta\omega_n x(0)}{\omega_a} \qquad B = x(0)$$

O bien,

$$x(t) = X_o e^{-\zeta\omega_n t}\,\mathbf{cos}(\omega_a t - \phi)\,, \quad X_o = \sqrt{\left(\frac{\dot{x}(0) + \zeta\omega_n x(0)}{\omega_a}\right)^2 + \left(x(0)\right)^2} \quad \phi = \mathbf{tan}^{-1}\frac{\dot{x}(0) + \zeta\omega_n x(0)}{x(0)\omega_a}$$

Decremento logarítmico $\quad \Delta = \mathbf{ln}\dfrac{x_i}{x_j} = \dfrac{2\pi n\zeta}{\sqrt{1-\zeta^2}} \qquad \zeta = \dfrac{\Delta}{\sqrt{(2\pi n)^2 + \Delta^2}} \qquad x_i > x_j$

Sistema Críticamente amortiguado: $\zeta = 1$

$$x(t) = \left[A + Bt\right]e^{-\zeta\omega_n t} \qquad A = x(0) \qquad B = \dot{x}(0) + \omega_n x(0)$$

Sistema Sobreamortiguado: $\zeta > 1$

$$x(t) = Ae^{(-\zeta+\sqrt{\zeta^2-1})\omega_n t} + Be^{(-\zeta-\sqrt{\zeta^2-1})\omega_n t} \quad A = \frac{\dot{x}(0) + (\zeta + \sqrt{\zeta^2-1})\omega_n x(0)}{2\omega_n\sqrt{\zeta^2-1}} \quad B = \frac{-\dot{x}(0) - (\zeta - \sqrt{\zeta^2-1})\omega_n x(0)}{2\omega_n\sqrt{\zeta^2-1}}$$

VIBRACIONES FORZADAS EN REGIMEN PERMANENTE: $x(t) = x_p(t)$

Coef. de amplitud dinámico	Coef. de transmisibilidad:	Relación entre frecuencias
$K = \dfrac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$	$\tau = \dfrac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$	$r = \dfrac{\Omega}{\omega_n}$

Vibraciones por excitación armónica: $m_e\ddot{x} + c_e\dot{x} + k_e x = F_o \cos(\Omega t)$

$x(t) = X_m \cos(\Omega t - \varphi)$ $\quad X_m = \dfrac{F_o}{k_e}K$ $\quad \tan\varphi = \dfrac{2\zeta r}{1-r^2}$ $\quad r_{crit} = \sqrt{1-2\zeta^2}$

Fuerza transmitida a la Fundación: $F_T = c\dot{x}(t) + kx(t)$

$F_T = F_{T\max}\cos(\Omega t - \varphi - \phi)$ $\quad F_{T\max} = F_o\tau$ $\quad \tan\varphi = \dfrac{2\zeta r}{1-r^2}$ $\quad \tan\phi = 2\zeta r$

Vibraciones por masa desbalanceada: $M_e\ddot{x} + c_e\dot{x} + k_e x = me\Omega^2 \cos(\Omega t)$

$x(t) = X_m \cos(\Omega t - \varphi)$ $\quad X_m = \dfrac{me\Omega^2}{k_e}K = \dfrac{me}{M_e}r^2K$ $\quad \tan\varphi = \dfrac{2\zeta r}{1-r^2}$ $\quad r_{crit} = \dfrac{1}{\sqrt{1-2\zeta^2}}$

Fuerza transmitida a la Fundación: $F_T = c\dot{x}(t) + kx(t)$

$F_T = F_{T\max}\cos(\Omega t - \varphi - \phi)$ $\quad F_{T\max} = F_o\tau$ $\quad F_{T\max} = me\Omega^2\tau = me\omega_n^2 r^2\tau$

Vibraciones por centro de masa desbalanceado: $M_e\ddot{x} + c_e\dot{x} + k_e x = M_e e\Omega^2 \cos(\Omega t)$

$x(t) = X_m \cos(\Omega t - \varphi)$ $\quad X_m = \dfrac{M_e e\Omega^2}{k_e}K$ $\quad X_m = er^2K$

Fuerza transmitida a la Fundación:

$F_T = c\dot{x}(t) + kx(t)$ $\quad F_T = F_{T\max}\cos(\Omega t - \varphi - \phi)$ $\quad F_{T\max} = M_e e\omega_n^2 r^2\tau$

Vibraciones por movimiento de la base: $Y = Y_m \cos(\Omega t)$**:**

Coor.absoluta x:

$m_e\ddot{x} + c_e(\dot{x} - \dot{y}) + k_e(x - y) = 0$ $\quad m_e\ddot{x} + c_e\dot{x} + k_e x = k_e y + c_e\dot{y}$

Coor. relativa z:

$m_e\ddot{x} + c_e(\dot{x} - \dot{y}) + k_e(x - y) = 0$ $\quad m_e\ddot{z} + c_e\dot{z} + k_e z = -m_e\ddot{y}$

VIBRACIONES TRANSITORIAS: $x(t) = x_h(t) + x_p(t)$

Si se utiliza Laplace: $x(t) = L^{-1}\left[H(s)F(s)\right] + x_{CI}(t)$ con $H(s) = \dfrac{1}{(ms^2 + cs + k)}$

Vibraciones por fuerzas no periódicas:

Respuesta a un impulso unitario $f(t) = \delta(t)$ $F(s) = 1$
$h(t) = \dfrac{1}{m\omega_a} e^{-\zeta\omega_n t} \mathbf{sin}\,\omega_a t$ si $\zeta = 0$ $h(t) = \dfrac{1}{m\omega_n} \mathbf{sin}\,\omega_n t$

Respuesta a un escalón unitario $f(t) = u(t)$ $F(s) = 1/s$
$g(t) = \dfrac{1}{k}\left[1 - e^{-\zeta\omega_n t}\left(\mathbf{cos}\,\omega_a t + \dfrac{\zeta}{\sqrt{1-\zeta^2}}\mathbf{sin}\,\omega_a t\right)\right]$ si $\zeta = 0$ $g(t) = \dfrac{1}{k}\left[1 - \mathbf{cos}\,\omega_n t\right]$

Respuesta a una rampa de pendiente unitaria, $f(t) = t$ $F(s) = 1/s^2$
$r(t) = \dfrac{1}{k}\left[t - \dfrac{2\zeta}{\omega_n} + e^{-\zeta\omega_n t}\left(\dfrac{2\zeta}{\omega_n}\mathbf{cos}(\omega_a t) + \dfrac{2\zeta^2 - 1}{\omega_a}\mathbf{sin}(\omega_a t)\right)\right]$ si $\zeta = 0$ $r(t) = \dfrac{1}{k}\left[t - \dfrac{1}{\omega_n}\mathbf{sin}(\omega_n t)\right]$

Respuesta a las Condiciones Iniciales:
$x_{CI}(t) = e^{-\zeta\omega_n t}\left(\dfrac{\dot{x}(0) + \zeta\omega_n x(0)}{\omega_a}\mathbf{sen}(\omega_a t) + x(0)\,\mathbf{cos}(\omega_a t)\right)$

Integral de Convolución:	**Integral de Duhamel:**
$x(t) = \int_0^t f(\tau)h(t-\tau)d\tau$ $x(t) = \int_0^t f(t-\tau)h(\tau)d\tau$	$x(t) = f(0)g(t) + \int_0^t f'(\tau)g(t-\tau)d\tau$